m-vector spin glass in the presence of a Gaussian random field

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The infinite-range m-vector spin glass in the presence of a Gaussian random field (characterized by a mean h_0 and width Δ) is investigated through the replica method. The phase diagram of the model, for $h_0=0$ and Δ finite, is obtained within replica-symmetry approximation. It is shown that the paramagnetic phase is enhanced, contrary to what happens to the spin-glass, mixed-ferromagnetic, and ferromagnetic phases, for growing values of Δ. For $h_0\neq 0$, the changes introduced in the Gabay-Toulouse line by finite values of Δ are investigated. In particular, it is shown that the critical exponent characteristic of the Gabay-Toulouse line is nonuniversal, changing continuously with the distribution width Δ. [S1063-651X(96)06205-8]

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I. INTRODUCTION

Disordered magnetic systems became, in the past decade or so, one of the most promising and exciting areas of physics. Among them, ferromagnets in the presence of random fields and spin glasses have attracted the attention of many workers.

The random-field problem has gained a lot of interest after the revelation of its physical realization as a diluted Ising antiferromagnet in the presence of a uniform magnetic field along the uniaxial direction [1] and that the static critical properties in these two systems may be the same [2]. Since then, a lot of effort has been dedicated to such systems, from both theoretical and experimental points of view [3,4]. Most of the theoretical work is based on the Ising model. Although the Ising condition (i.e., the magnetic spins restricted to a single direction) is never strictly satisfied in experimental systems, materials with uniaxial anisotropy will normally exhibit Ising behavior, allowing a confrontation between theory and experiment. Among interesting questions related to this problem, one may single out how continuous m-vector spin systems shall behave in the presence of random fields and which properties they shall have in common with spin glasses [4,5].

Since the proposal of Edwards and Anderson [6] of a model to describe spin glasses [7,8], this subject became one of the most puzzling and controversial topics in statistical physics. Today, the existence of a spin-glass phase for the short-range three-dimensional Ising spin glass [9] is well accepted and its lower critical dimension d_l lies in the range $2<d_l<3$. However, the understanding of the nature of such a phase is the point of an exciting dispute. The predictions of the Ising mean-field theory, based on the infinite-range model proposed by Sherrington and Kirkpatrick [10], are astonishing: the spin-glass phase is characterized by replica-symmetry breaking, being properly described by an order-parameter function [11], i.e., an infinite number of order parameters, which are organized in a hierarchical structure, within an ultrametric space [12]. Another surprising result is the existence of a phase transition in the presence of a uniform magnetic field, signaled by the Almeida-Thouless (AT) line [13]. The rival picture [14,15], called the droplet model, is based on renormalization-group ideas, assuming that the spin-glass phase is of the trivial sort, i.e., described by a single order parameter. Their predictions differ radically from those based on the Sherrington-Kirkpatrick (SK) model, concluding that for *any finite dimension*, there is no ultrametricity, or AT line. Although a lot of effort has been dedicated to this question [16–26], a definitive answer is still missing. Numerical works done on hypercubic lattices with dimensions $d>3$ [18–22] find a picture compatible with mean-field theory, suggesting the failure of the droplet model. However, for $d=3$ the situation remains unclear [25,26].

Much less is known for the continuous m-vector spin glasses. Most of the numerical works done so far, for Heisenberg spins ($m=3$) in three dimensions, suggest that there is no isotropic spin-glass order [27–29], although a small anisotropy is sufficient to induce a phase transition [28,30]. In four dimensions, there is numerical evidence of a phase transition [31] and so the lower critical dimension d_l for the short-range Heisenberg spin glass should satisfy $3<d_l<4$. For XY spins ($m=2$) in three dimensions, two equally acceptable fits corresponding, respectively, to zero- and finite-temperature phase transitions were found [32]; however, the defect-energy method [28] gives no phase transition for the three-dimensional XY spin glass. Although one would expect the same lower critical dimension for both $m=2$ and 3 cases, such as in ferromagnetic systems, this should not necessarily hold. The m-vector mean-field solution is also based on an infinite-range model, presenting features similar to the SK model [33]. In the presence of a uniform magnetic field, there is a phase transition associated with the transverse degrees of freedom, signaled by the Gabay-Toulouse (GT) line [34]. Below this line, replica-symmetry breaking is necessary [35] for a proper description of the spin-glass phase. The GT line has been observed recently in the Heisenberg spin glass CuMn [36], giving support to its mean-field theory.

The combination of spin-glass and random-field models has been successful for the description of diluted antiferromagnets [37], mixed hydrogen-bonded ferroelectrics, and antiferroelectrics, i.e., proton and deuteron glasses [38] and orientational glasses [39]. The infinite-range Ising spin-glass model in the presence of a Gaussian random field [37] was able to describe several experimental results obtained for the diluted antiferromagnet $Fe_xZn_{1-x}F_2$ ($0.25 \leqslant x \leqslant 0.40$) [40]. In particular, for a small width of the field distribution, a crossover between the random-field and spin-glass regimes was verified [37], which is in good agreement with the experiments done on $Fe_{0.31}Zn_{0.69}F_2$ [40,41].

In this paper we consider the infinite-range m-vector spin glass in the presence of a Gaussian random field. Apart from a generalization of the Ising case [37], it represents a very relevant system from the experimental point of view. We investigate how the presence of the random field affects the well-known phase diagrams of the m-vector spin glasses and in particular how the GT line is modified by a finite-width distribution for the magnetic field. In Sec. II we define the model and make use of the replica method to find its free-energy functional. In Sec. III we present the phase diagrams and the corresponding analytical results. Finally, in Sec. IV we present our conclusions.

II. THE MODEL AND ITS FREE-ENERGY DENSITY

Let us consider the m-vector spin glass in the presence of an external random magnetic field (favoring the $\vec{1}$ direction), defined through the Hamiltonian

$$\mathscr{H} = -\sum_{i,j} J_{ij} \vec{S}_i \cdot \vec{S}_j - \sum_i h_i S_{i1}, \tag{2.1}$$

where $\vec{S}_i$ ($i=1,2,\ldots,N$) are classical m-component spin variables

$$\vec{S}_i \equiv (S_{i1}, S_{i2}, \ldots, S_{im}), \quad \sum_{\mu=1}^{m} S_{i\mu}^2 = m. \tag{2.2}$$

The interactions are infinite-range-like, i.e., the sum $\Sigma_{i,j}$ extends over all distinct pairs of spins; the coupling constants $\{J_{ij}\}$ and the fields $\{h_i\}$ are quenched random variables, following independent Gaussian probability distributions

$$P(J_{ij}) = \left(\frac{N}{2\pi J^2}\right)^{1/2} \exp\left[-\frac{N}{2J^2}\left(J_{ij} - \frac{J_0}{N}\right)^2\right], \tag{2.3}$$

$$P(h_i) = \left(\frac{1}{2\pi\Delta^2}\right)^{1/2} \exp\left[-\frac{1}{2\Delta^2}(h_i - h_0)^2\right]. \tag{2.4}$$

For a given realization of the disorder (bonds $\{J_{ij}\}$ and fields $\{h_i\}$), one has the corresponding free energy $F(\{J_{ij}\},\{h_i\})$; for quenched systems, the average over the disorder $[\;]_{J,h}$ should be taken over the free energy

$$[F(\{J_{ij}\},\{h_i\})]_{J,h} = \int \prod_{i,j} [dJ_{ij} P(J_{ij})] \prod_i [dh_i P(h_i)] \times F(\{J_{ij}\},\{h_i\}). \tag{2.5}$$

As usual, this average is computed by means of the replica trick [7,8]; one gets the free energy per spin

$$-\beta f = \lim_{N\to\infty} \frac{1}{N} [\ln Z(\{J_{ij}\},\{h_i\})]_{J,h} = \lim_{N\to\infty} \lim_{n\to 0} \frac{1}{Nn} ([Z^n]_{J,h} - 1), \tag{2.6}$$

where Z^n is the partition function of n replicas of the system defined through (2.1). The standard procedure leads to

$$\beta f = \lim_{n\to 0} \frac{1}{n} \min g(x^\alpha, M_1^\alpha, q_1^{\alpha\beta}, q^{\alpha\beta}), \tag{2.7}$$

where

$$\begin{aligned} g(x^\alpha, M_1^\alpha, q_1^{\alpha\beta}, q^{\alpha\beta}) &= -\tfrac{1}{4}(\beta J)^2 nm + \tfrac{1}{2}(\beta J)^2 \sum_\alpha \left[\frac{m(m-1)}{2}(x^\alpha)^2 + m x^\alpha\right] \\ &+ \frac{\beta J_0}{2} \sum_\alpha (M_1^\alpha)^2 + \tfrac{1}{2}(\beta J)^2 \sum_{\alpha,\beta} (q_1^{\alpha\beta})^2 + \tfrac{1}{2}(m-1) \\ &\times (\beta J)^2 \sum_{\alpha,\beta} (q^{\alpha\beta})^2 - \ln \mathrm{Tr}_\alpha \exp(\mathscr{H}_{\mathrm{eff}}), \end{aligned} \tag{2.8a}$$

$$\begin{aligned} \mathscr{H}_{\mathrm{eff}} &= \tfrac{1}{2}(\beta\Delta)^2 \sum_\alpha (S_1^\alpha)^2 + \tfrac{1}{2} m (\beta J)^2 \sum_\alpha x^\alpha (S_1^\alpha)^2 \\ &+ \beta J_0 \sum_\alpha M_1^\alpha S_1^\alpha + (\beta\Delta)^2 \sum_{\alpha,\beta} S_1^\alpha S_1^\beta \\ &+ (\beta J)^2 \sum_{\alpha,\beta} q_1^{\alpha\beta} S_1^\alpha S_1^\beta + (\beta J)^2 \sum_{\alpha,\beta} \sum_{\mu\neq 1} q^{\alpha\beta} S_\mu^\alpha S_\mu^\beta \\ &+ \beta h_0 \sum_\alpha S_1^\alpha. \end{aligned} \tag{2.8b}$$

In the equations above, α and β ($\alpha,\beta = 1,2,\ldots,n$) are replica labels μ ($\mu = 1,2,\ldots,m$) denote Cartesian components, Tr_α is a trace over the spin variables for each replica α, and $\Sigma_{\alpha,\beta}$ represent sums over distinct pairs of replicas. The extrema of the functional $g(x^\alpha, M_1^\alpha, q_1^{\alpha\beta}, q^{\alpha\beta})$ give us the equations of state

$$(m-1)x^\alpha = \langle (S_1^\alpha)^2 \rangle - 1, \tag{2.9a}$$

$$M_1^\alpha = \langle S_1^\alpha \rangle, \tag{2.9b}$$

$$q_1^{\alpha\beta} = \langle S_1^\alpha S_1^\beta \rangle \quad (\alpha \neq \beta), \tag{2.9c}$$

$$q^{\alpha\beta} = \frac{1}{m-1} \sum_{\mu\neq 1} \langle S_\mu^\alpha S_\mu^\beta \rangle \quad (\alpha \neq \beta), \tag{2.9d}$$

where the angular brackets denote thermal averages with respect to the "effective Hamiltonian" $\mathscr{H}_{\mathrm{eff}}$ in (2.8b).

The analytic continuation $n \to 0$ in (2.7) may be easily carried (see the Appendix) if one considers the replica-symmetry approximation [10]

$$x^\alpha = x, \quad M_1^\alpha = M_1 \quad \forall \alpha$$
$$q_1^{\alpha\beta} = q_1, \quad q^{\alpha\beta} = q \quad \forall \alpha, \beta. \tag{2.10}$$

The region in the phase diagram where this replica-symmetric solution is valid is obtained through the stability analysis of (2.7) around (2.10). Using the ansatz (2.10), the replica-symmetric phase diagram for continuous m-vector spins may be constructed [34,35]. In the case of no external magnetic field and $J_0>0$, the relevant parameters for the description of the system are M_1, q_1, and q [33,34], whereas if $J_0=0$, the presence of a uniform magnetic field (in the $\vec{1}$ direction) induces the parameters x, M_1, and q_1, allowing for a phase transition in the transverse spin-glass order parameter q, signaled by the Gabay-Toulouse line [34]. Analogous to the Almeida-Thouless line [13] for the Ising case, the GT line is also associated with replica-symmetry breaking [35], playing a similar role for m-vector spin glasses. Now, in the case of a random external magnetic field, as defined by (2.4), the width of the distribution ($\Delta \neq 0$) induces the parameters x and q_1. Therefore, if $h_0=0$ and $J_0>0$, the relevant parameters are M_1 and q, whereas if $J_0=0$ and $h_0>0$, there is still a GT line in the plane h_0 versus temperature. The changes in the GT line, as well as in the phase diagram for $J_0>0$, due to the Gaussian random field of width Δ, is the subject we will discuss in the next section.

III. RESULTS

A. Case $J_0=0$

For the random-field distribution given in (2.4), with nonzero mean and width ($h_0,\Delta>0$), the only possible transition is the one associated with the transverse spin-glass order parameter q. The Gabay-Toulouse line is obtained by solving the set of equations (A9)–(A11) (see the Appendix).

Analytical results for the GT line may be obtained in two regimes, namely, low fields ($h_0,\Delta \ll J$), as well as high fields and low temperatures ($h_0 \gg J$, $T \ll J$, and any Δ). In the first case, Eqs. (A9)–(A11) may be expanded in power series to give the induced parameters

$$q_1 \cong \frac{1}{\sqrt{2}} \left[\left(\frac{h_0}{J} \right)^2 + \left(\frac{\Delta}{J} \right)^2 \right]^{1/2}, \quad x \cong \frac{1}{4} \left[\left(\frac{h_0}{J} \right)^2 + \left(\frac{\Delta}{J} \right)^2 \right] \tag{3.1}$$

and the GT line

$$\frac{T}{J} \cong \frac{\tilde{T}}{J} - \frac{m^2+4m+2}{4(m+2)^2} \left(\frac{h_0}{J} \right)^2, \tag{3.2}$$

where $\tilde{T}$ denotes the temperature at which the GT line meets the axis $h_0=0$,

$$\frac{\tilde{T}}{J} \cong 1 - \frac{m^2+4m+2}{4(m+2)^2} \left(\frac{\Delta}{J} \right)^2. \tag{3.3}$$

For high fields and low temperatures, one gets the usual exponential decay

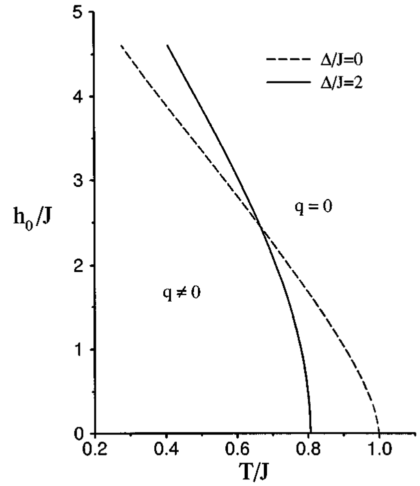

FIG. 1. Gabay-Toulouse line for the m-vector spin glass in the presence of a Gaussian random field (mean h_0 and width Δ; $\Delta/J=2$). The dashed curve represents the GT line in the case of a uniform field ($\Delta/J=0$).

$$\frac{T}{J} \cong \left(\frac{2}{\pi} \right)^{1/2} \frac{1+m}{[m+(\Delta/J)^2]^{1/2}} \exp\left\{ -\frac{(h_0/J)^2}{2[m+(\Delta/J)^2]} \right\}. \tag{3.4}$$

For all other situations, Eqs. (A9)–(A11) are solved numerically. The search for the solutions of such nonlinear equations was done through IMSL Subroutine ZSYSTM, whereas all integrals were evaluated by using Simpson's rule. Although the Gaussian integrals are defined from $-\infty$ to $+\infty$, we have chosen the limits -4 and $+4$; apart from avoiding the numerical difficulties present in broader ranges, this choice captures approximately 99.98% of the total area in the corresponding integral. The inner integrals [functions P_{nk} in Eq. (A12)] present, within Simpson's rule, a better convergence whenever the integration limits are integer numbers; our numerical analysis was carried for $m=4$, in which case such limits are -2 and $+2$. This choice does not change the qualitative behavior of our phase diagrams, as will be discussed at the end of this section. In both cases (Gaussian integrals and functions P_{nk}), the corresponding range of integration was divided into 400 segments for the application of Simpson's rule. A solution was accepted when the left- and right-hand sides of all three equations agreed within differences less than or equal to 10^{-3}.

In Fig. 1 we present the GT line for a typical value of Δ ($\Delta/J=2$). One sees that the low-field part of the GT line moves to the left as Δ increases, i.e., the width of the distribution favors the longitudinal degrees of freedom for $h_0 \ll J$ and so the transverse ordering becomes more difficult. This fact is seen analytically, where the temperature $\tilde{T}$ [Eq. (3.3)] is shifted by the introduction of a small width Δ. One also observes that the exponential decay, for high fields and low temperatures [Eq. (3.4)], is weakened by a finite width Δ. The strongly polarized state, characterized by the prevalence of the longitudinal degrees of freedom, gets diminished by the presence of a finite Δ. Whereas a large mean h_0 together with a small temperature ($h_0 \gg J$ and $T \ll J$) produce a freezing of spins in the positive $\vec{1}$ direction, the width Δ allows for

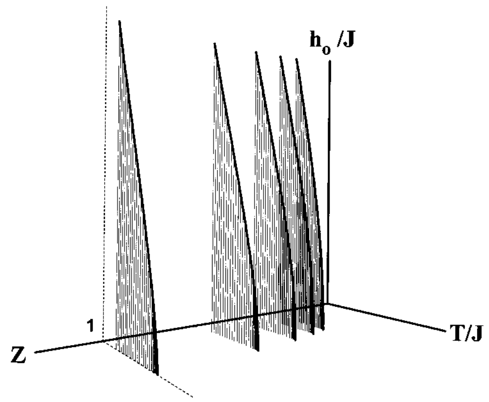

FIG. 2. Evolution of the Gabay-Toulouse line as Δ varies $[z=1/(1+\Delta/J)]$.

fluctuations of the spins along the $\vec{1}$ axis in both positive and negative directions. Such fluctuations produce a weakening in the longitudinal degrees of freedom. Therefore, the overall effect of the width Δ in the GT line is to enhance the longitudinal (transverse) degrees of freedom in the low-mean (high-mean and low-temperature) regime. This explains the intersection of the two GT lines (corresponding to $\Delta/J=0$ and 2) exhibited in Fig. 1.

In Fig. 2 we present a three-dimensional plot representing the evolution of the GT line with Δ. As discussed above, the low- and high-field parts of the curve move oppositely as Δ departs from zero. As Δ becomes large, the GT line tends to a straight line, parallel to the h_0/J axis, joining this axis as $\Delta\to\infty$. The sector associated with replica-symmetry breaking (phase to the left of the GT line) decreases for increasing Δ.

In the low-mean regime ($h_0 \ll J$), one may define the critical exponent ϕ,

$$\frac{\tilde{T}}{J}-\frac{T}{J}\sim\left(\frac{h_0}{J}\right)^{\phi}. \tag{3.5}$$

A uniform field ($\Delta=0$) yields $\tilde{T}/J=1$ and $\phi=2$ [34]; if a small width Δ is introduced, our lowest-order calculation shows that $\tilde{T}/J$ is shifted to the left [Eq. (3.3)], whereas the exponent ϕ remains unchanged [Eq. (3.2)]. However, Figs. 1 and 2 show clearly that the exponent ϕ changes for higher values of Δ. We computed the exponent ϕ for increasing values of Δ and verified its *nonuniversal character*; in fact, it varies continuously with Δ [$\phi=\phi(\Delta)$]. This aspect is presented in Fig. 3, where the exponent ϕ is shown to vary continuously between $\phi=2$ (for $\Delta=0$) to $\phi=1$ (for $\Delta\to\infty$). The breakdown of universality has already been found in Ising spin glasses for both infinite- [37] and short-range [42] interactions. In the latter case it was shown that the shape of the $\{J_{ij}\}$ probability distribution influences the critical exponents.

B. Case $h_0=0$

A finite random-field width ($\Delta\neq 0$) induces the parameters x and q_1 such that the relevant parameters are now M_1 and q. Four phases are possible in this case, namely, the paramagnetic (P) ($M_1=q=0$), the spin-glass (SG) ($M_1=0$, $q\neq 0$), the ferromagnetic (F) ($M_1\neq 0$, $q=0$), and the mixed-

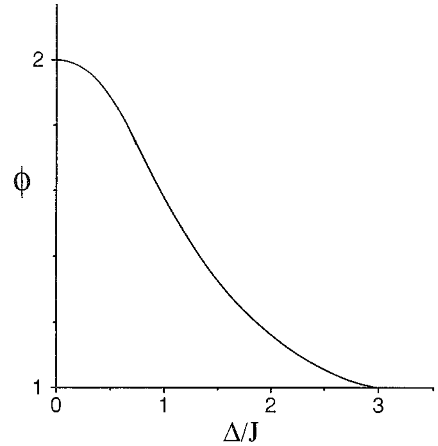

FIG. 3. The exponent ϕ characteristic of the Gabay-Toulouse line in the low-mean regime ($h_0 \ll J$) is nonuniversal, depending on the width of the random field (Δ).

ferromagnetic (F') ($M_1\neq 0$, $q\neq 0$) phases. The F' phase was encountered by Gabay and Toulouse [34] and is characterized by the coexistence of both ferromagnetic and transverse spin-glass orderings.

As usual, the equilibrium equations [Eqs. (A6)] present the symmetry $h_0\to J_0 M_1$ and so some of the results obtained in the previous case may be trivially translated to the present one. As an example, the phase transition P-SG occurs at the same temperature $\tilde{T}$, at which the GT line meets the axis $h_0=0$ in Sec. III A [see Eq. (3.3)].

Let us restrict ourselves, for the moment, to small random-field widths ($\Delta \ll J$). The multicritical point, where all four critical lines meet, is located at

$$\frac{\tilde{J}_0}{J}\cong 1+\frac{1}{\sqrt{2}}\frac{\Delta}{J}, \quad \frac{\tilde{T}}{J}\cong 1-\frac{m^2+4m+2}{4(m+2)^2}\left(\frac{\Delta}{J}\right)^2. \tag{3.6}$$

This shows that a finite random-field width favors the paramagnetic phase (with induced parameters x and q_1); all other phases are reduced due to $\Delta>0$. The critical frontier P-F deviates from the straight line characteristic of the case $\Delta=0$ [34]; indeed, near the multicritical point it is given by

$$\frac{T}{J}\cong\frac{J_0}{J}-\frac{1}{\sqrt{2}}\frac{\Delta/J}{J_0/J}, \tag{3.7}$$

whereas in the high-temperature limit one finds

$$\frac{T}{J}\cong\frac{J_0}{J}-\left[\frac{3}{m+2}+\frac{(J_0/J)^2}{(J_0/J)^2-1}\right]\frac{(\Delta/J)^2}{J_0/J}. \tag{3.8}$$

The phase transition associated with the transverse degrees of freedom [similar to the GT line of Sec. III A] is the critical frontier F-F'. Near the multicritical point, this line is given by

$$\frac{\tilde{T}}{J}-\frac{T}{J}\cong\frac{m^2+4m+2}{2(m+2)^2}\left[\left(\frac{J_0}{J}-1\right)^2-\left(\frac{\tilde{J}_0}{J}-1\right)^2\right], \tag{3.9}$$

whereas in the $J_0\to\infty$ limit, one gets a similar exponential decay [cf. Eq. (3.4)]

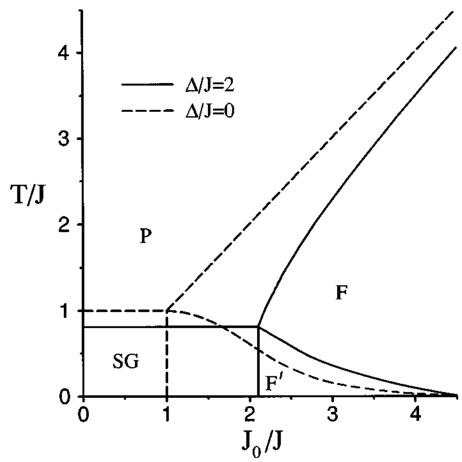

FIG. 4. Phase diagram of the m-vector spin glass in the presence of a Gaussian random field of width Δ ($\Delta/J=2$), showing the paramagnetic (P), ferromagnetic (F), spin-glass (SG), and mixed-ferromagnetic (F') phases. The dashed lines represent the critical frontiers in the case $\Delta/J=0$.

$$\frac{T}{J}\cong\left(\frac{2}{\pi}\right)^{1/2}\frac{1+m}{[m+(\Delta/J)^2]^{1/2}}\exp\left\{-\frac{m(J_0/J)^2}{2[m+(\Delta/J)^2]}\right\}, \tag{3.10}$$

which is valid for any Δ.

For all other situations, the critical frontiers are obtained by numerically solving the set of equations (A6) using the same procedure applied for the case $J_0=0$ [Eqs. (A9)–(A11)], as described previously. For the SG-F' critical frontier, we have adopted the Parisi-Toulouse hypothesis [43], according to which such a frontier should be a vertical straight line. The phase diagram of the m-vector spin glass in the presence of a Gaussian random field (mean $h_0=0$, width $\Delta/J=2$) is shown in Fig. 4. By comparing it with the case $\Delta/J=0$ [33,34] one concludes the following.

(i) The paramagnetic portion (P) of the phase diagram, characterized by the induced parameters x and q_1, increases due to the random field; this is a consequence of the fact that such induced parameters make it harder for the ferromagnetic orderings (F and F') and spin glass to occur.

(ii) The critical frontier P-F is no longer a straight line, as already shown for small values of Δ [cf. Eqs. (3.7) and (3.8)].

(iii) The critical frontier F-F' near the multicritical point is expected to behave as

$$\frac{\widetilde{T}}{J}-\frac{T}{J}\sim\left(\frac{J_0}{J}-\frac{\widetilde{J}_0}{J}\right)^{\phi'}. \tag{3.11}$$

Similarly to what was found for the GT line in the previous case ($J_0=0$), one should have a nonuniversal exponent $\phi'=\phi'(\Delta)$. This is clearly observed in the two corresponding critical frontiers shown in Fig. 4 [$\Delta/J=0$ (dashed) and $\Delta/J=2$ (full line)].

(iv) The sector of the phase diagram associated with replica-symmetry breaking (phases SG and F') get diminished as Δ increases, similar to what happens for the Ising spin glass in a random field [37].

Finally, it should be said that our phase diagrams were obtained for $m=4$, whereas the most interesting physical situation corresponds to $m=3$ (Heisenberg spins). The choice $m=4$ was a mere numerical convenience, but the qualitative behavior of our results should hold for any finite m ($m\geqslant 2$), such as in the case of a uniform magnetic field [34,35], where no new qualitative features are expected for varying m. Although the location of the critical frontiers may change with the value of m, e.g., the temperature at which the GT line meets the axis $h_0=0$ in Fig. 1, or the position of the multicritical point in Fig. 4, as already seen for the case $\Delta\ll J$ [cf. Eqs. (3.3) and (3.6)], the qualitative shapes of the critical frontiers remain unaltered. As a consequence of this, the critical exponents do not change. In particular, the nonuniversal exponent $\phi(\Delta)$, shown in Fig. 3, is expected to vary continuously in the range from $\phi=2$ (for $\Delta=0$ [34]) to $\phi=1$ (for $\Delta\to\infty$, at which limit the GT line, for $\widetilde{T}/J\to 0$, should meet the h_0/J axis as a vertical straight line in Fig. 2), for all values of m.

IV. CONCLUSION

We have studied the m-vector spin glass in the presence of a Gaussian random field of mean h_0 and width Δ. The phase diagrams in the cases $h_0\neq 0$ and $h_0=0$ were analyzed for different values of Δ. It was shown that the sectors of the phase diagrams associated with replica-symmetry breaking decrease as Δ increases; a similar effect has already been observed for the Sherrington-Kirkpatrick model in the presence of a Gaussian random field [37]. It was observed that one of the effects produced by a finite width Δ on the Gabay-Toulouse line is to enhance the longitudinal degrees of freedom in the low-field regime (h_0 small), whereas in the high-field regime (h_0 large), the opposite behavior was verified, i.e., the longitudinal degrees of freedom were weakened by the presence of the random field. In both types of phase diagrams, a nonuniversal behavior was found, either in the GT line ($h_0\neq 0$) or in the ferromagnetic–mixed-ferromagnetic ($h_0=0$) critical frontier. To our knowledge, this is the first time that a breakdown of universality has been reported for m-vector spin glasses. For the moment, we are not aware of any measurements to support our results. We believe that Heisenberg-like diluted antiferromagnets are good candidates; they should, at least qualitatively, present some of the characteristics predicted in the present work.

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APPENDIX

In this appendix we consider the m-vector spin glass, as defined through (2.1), within the replica-symmetry approximation; the equations and notation herein closely follow those of Ref. [35]. Considering choice (2.10), the free energy per spin [Eq. (2.7)] becomes

$$\beta f=-\tfrac{1}{4}(\beta J)^2 m+\tfrac{1}{2}(\beta J)^2\left[\frac{m(m-1)}{2}x^2+mx\right]+\frac{\beta J_0}{2}M_1^2 -\tfrac{1}{4}(\beta J)^2 q_1^2-\tfrac{1}{4}(m-1)(\beta J)^2 q^2+\tfrac{1}{2}m(\beta J)^2 q -\int_{-\infty}^{\infty}\prod_{\mu=1}^{m}\frac{dv_\mu}{(2\pi)^{1/2}}\exp\left(-\sum_{\mu=1}^{m}v_\mu^2/2\right)\ln\tilde{Z}, \tag{A1}$$

where

$$\tilde{Z}=\mathrm{Tr}\exp\left(\sum_{\mu=1}^{m}a_\mu S_\mu+bS_1^2\right), \tag{A2}$$

$$a_1=\beta J_0 M_1+\beta J\left[q_1+\left(\frac{\Delta}{J}\right)^2\right]^{1/2}v_1+\beta h_0, \tag{A3}$$

$$a_\mu=\beta J q_\mu^{1/2}v_\mu,\quad \mu\neq 1 \tag{A4}$$

$$b=\tfrac{1}{2}(\beta J)^2[mx-(q_1-q)]. \tag{A5}$$

The parameters in Eqs. (2.9) may be expressed in terms of $\tilde{Z}$,

$$(m-1)x+1=\left\langle\frac{1}{\tilde{Z}}\frac{\partial^2\tilde{Z}}{\partial a_1^2}\right\rangle_v, \tag{A6a}$$

$$M_1=\left\langle\frac{1}{\tilde{Z}}\frac{\partial\tilde{Z}}{\partial a_1}\right\rangle_v, \tag{A6b}$$

$$q_1=\left\langle\left(\frac{1}{\tilde{Z}}\frac{\partial\tilde{Z}}{\partial a_1}\right)^2\right\rangle_v, \tag{A6c}$$

$$q=\frac{1}{m-1}\sum_{\mu\neq 1}\left\langle\left(\frac{1}{\tilde{Z}}\frac{\partial\tilde{Z}}{\partial a_\mu}\right)^2\right\rangle_v, \tag{A6d}$$

where $\langle\ \rangle_v$ represents the Gaussian averages that appear in (A1). The trace in (A2) is an integral over an m-dimensional hypersphere and may be expressed as

$$\tilde{Z}=\tfrac{1}{2}(2\pi)^{(m-1)/2}r^{(3-m)/2}\int_{-\sqrt{m}}^{\sqrt{m}}dS_1\exp(a_1S_1+bS_1^2) \times(m-S_1^2)^{(m-3)/4}I_{(m-3)/2}(r(m-S_1^2)^{1/2}), \tag{A7}$$

where

$$r=(a_2^2+a_3^2+\cdots+a_m^2)^{1/2} \tag{A8}$$

and $I_k(z)$ are modified Bessel functions of the first kind, of order k.

The Gabay-Toulouse line, which signals the onset of the transverse degrees of freedom, is obtained from Eqs. (A6) in the limit $q\to 0$,

$$(m-1)^2\left(\frac{T}{J}\right)^2=\int_{-\infty}^{\infty}\frac{dv_1}{(2\pi)^{1/2}}\exp(-v_1^2/2)(P_{20}/P_{00})^2, \tag{A9}$$

$$1+(m-1)x=\int_{-\infty}^{\infty}\frac{dv_1}{(2\pi)^{1/2}}\exp(-v_1^2/2)[m-(P_{20}/P_{00})], \tag{A10}$$

$$q_1=\int_{-\infty}^{\infty}\frac{dv_1}{(2\pi)^{1/2}}\exp(-v_1^2/2)(P_{01}/P_{00})^2, \tag{A11}$$

where

$$P_{nk}=\int_{-\sqrt{m}}^{\sqrt{m}}dS_1\exp(a_1S_1+b_0S_1^2)(m-S_1^2)^{(m-3+n)/2}S_1^k, \tag{A12}$$

with

$$a_1=\beta J\left[q_1+\left(\frac{\Delta}{J}\right)^2\right]^{1/2}v_1+\beta h_0, \tag{A13}$$

$$b_0=\tfrac{1}{2}(\beta J)^2(mx-q_1). \tag{A14}$$

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